

**The exam questions of  
«Theory of functions and functional analysis»**

1. Metric space. Definition and Examples.
2. Convergence in metric spaces. Properties of converging sequences.
3. Open and closed sets. Dense subsets. Separable spaces.
4. Complete metric spaces. The nested sphere theorem. Baire's theorem
5. Contraction Mappings. Fixed point theorem. Contraction mappings and differential equations, Integral equations.
6. Topological Spaces. Bases, Axioms of Countability. Axioms of separation.
7. Compact Spaces. Compactness in Metric Spaces. Arzela's theorem.
8. Linear spaces. Examples. Subspaces of linear space. Factor-spaces
- 9 Normed Linear Spaces. Examples. Subspaces of a normed linear space.
10. Equivalency of norm. countable basic. Complete system.
11. Euclidean Spaces Existence of an orthogonal basis. Orthogonalization.
12. Bessel's inequality. Closed orthogonal system. Complete Euclidean spaces. The Riesz-Fischer theorem.
13. Hilbert space. The isomorphism theorem. Subspaces. Orthogonal complements and direct sums..
14. Characterization of Euclidean spaces.  
Linear functional and its' properties. The Minkowski functional.
15. Continuous Linear Functionals on a normed spaces. The Hahn-Banach theorem for a normed linear space.
16. The Conjugate Space. General form of linear functionals. Corollaries of Hahn-Banach theorem . Weak convergence.
- 17 Linear operator and its' properties. Continuity and boundedness.
18. The norm of a linear continuous operator. Inverse and Adjoint Operators. Invertibility . The space  $L(E, E_1)$

19. The sequences of linear bounded operators. Uniformly convergence. Banach Steynhauss Theorem .
20. The adjoint operator in Hilbert space. Self-adjoint operators.
21. The spectrum of an operator. The resolvent of linear operators. Spectrum of continuos selfadjoint operators.
22. Completely Continuous Operators. Basic properties of completely continuous operators.
23. Completely continuous operators in Hilbert space. Hilbert-Schmidt Theorem. Theorem on spectrum of Completely Continuous Operators

## **L I T E R A T U R E**

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4. Walter Rudin. "Functional Analysis" McGraw-Hill.2006
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