

## EXAM QUESTIONS

1. Subject and goals of elementary mathematics.
2. Pascal's sign for dividing natural numbers.
3. Sign of division by 3 in the set of natural numbers.
4. Sign of division by 11 in the set of natural numbers.
5. Euclidean algorithm on the set of natural numbers.
6. Algorithm for division with remainder.
7. Classification of real numbers using continued fractions.
8. Calculating the root of a number using continued fractions.
9. Polynomials. Division of polynomials. Bezout's theorem.
10. Proof of the theorem on the integer root of a polynomial with integer coefficients.
11. Theorem on the integer root of a polynomial with integer coefficients.
12. Theorem on the existence of at least one real root of an algebraic equation of odd degree with real coefficients.
13. Viet's formulas for the roots of a polynomial.
14. Concepts of equivalence, consequence and disjunction of equations.
15. Theorems on the equivalence of equations (Theorems 1, 2).
16. Theorems on the equivalence of equations (Theorems 3, 4).
17. Equivalence of systems of equations. Consequences of this system of equations.
18. Methods for solving a system of equations (linear transformation).
19. Methods for solving a system of linear equations (Cramer's rule or determinants).
20. Methods for solving a system of equations (reducing this system into disjunctions of simpler systems).
21. Methods for solving a system of equations (substitution).
22. Statements about the equivalence of inequalities (statements 1,2,3).
23. Statements about the equivalence of inequalities (statements 4,5,6,7).
24. Solving inequalities using the interval method.
25. Irrational inequalities.
26. Proof of the theorem that any rational number can be expressed as a finite continued fraction.
27. Research on divisibility  $x^n \pm a^n$  by  $x \pm a$ , where  $n$  - is any natural number.
28. Concepts of equality and consequences of inequalities.
29. Solving equations with an absolute value sign.
30. Solving inequalities with an absolute value sign.
31. Symmetric equations of four degrees.
32. Equations of four degrees with additional conditions imposed on the coefficients.
33. Solving reciprocal equations of even order.
34. Solving reciprocal equations of odd order.
35. Arithmetic progression and its properties (property 1, property 2, property 3).
36. Arithmetic progression and its properties (property 4, property 5, property 7).
37. Arithmetic progression and its properties (property 2, property 5, property 6).
38. Geometric progression and its properties (property 1, property 2, property 3).
39. Geometric progression and its properties (property 4, property 5, property 6).
40. The sum of an infinite geometric progression, if  $|q| < 1$ .
41. Axioms of planimetry (axioms 11,12,13,14).
42. Axioms of planimetry (axioms 1,2,3,4,5).
43. Axioms of planimetry (axioms 15, 16, 17).

44. Axioms of planimetry (axioms 6,7,8,9,10).
45. Axioms of stereometry.
46. Conclusions from the axioms of stereometry.
47. Inscribed circles of a polygon.
48. Circumscribed circles of a polygon.
49. Proof of the theorem on the existence of excircles of a triangle.
50. Relationships between the radii of the inscribed and circumscribed circles of a triangle and the radii of its excircles.
51. Property of the centroid of a triangle.
52. Property of the center of the inscribed circle of a triangle.
53. Property of the circumcenter of a triangle.
54. Proof of the necessity of Ceva's theorem.
55. Proof of the sufficiency of Ceva's theorem.
56. Trigonometric form of Ceva's theorem.
57. The condition of perpendicularity of two opposite sides of a quadrilateral.
58. Condition for the perpendicularity of the diagonals of a quadrilateral.
59. Derivation of the formula for the length of the midline of a quadrilateral.
60. Derivation of the formula between the distances of the midpoints of the diagonals of a quadrilateral.
61. Theorem on the segment connecting the midpoints of the midlines and diagonals of a quadrilateral.
62. Basic metric relations in a quadrilateral.
63. Theorem on cosines of quadrilaterals.
64. Bretschneider's theorem for quadrilaterals.
65. Corollaries of Bretschneider's theorem for quadrilaterals (Corollary 1, 2).
66. Corollaries from the formulas for the area of a quadrilateral (result 1, 2, 3).
67. Corollaries from the formulas for the area of a quadrilateral (result 4, 5, 6).
68. Property of the orthocenter of a triangle.
69. Proof of the formula for the scalar product of any vectors  $\overline{AB}$  and  $\overline{CD}$ :
 
$$2\overline{AB} \cdot \overline{CD} = AD^2 + BC^2 - AC^2 - BD^2 .$$
70. Derivation of the formula for the distance between the centers of the inscribed and circumscribed circles of a triangle.
71. Derivation of the formula for the area of a quadrilateral  $16S^2 = 4e^2 f^2 - (b^2 + d^2 - a^2 - c^2)^2$ , where  $a, b, c, d$  - are the sides of the quadrilateral, and  $e$  and  $f$  - are the diagonals of the quadrilateral.
72. Derivation of the formula for the area of a quadrilateral
 
$$S^2 = (p-a)(p-b)(p-c)(p-d) - abcd \cos^2 \frac{A+C}{2}$$
, where  $a, b, c, d$  - are the sides of the quadrilateral,  $p$  - are the semi-perimeter of the quadrilateral,  $A$  and  $C$  - are the opposite internal angles of the quadrilateral.