The absence of positive global periodic solution of a second-order semi linear parabolic equation with time-periodic coefficients

Shirmayil G. Bagyrov

Baku State University, Azerbaijan Institute of Mathematics and Mechanics National Academy of Sciences of Azerbaijan

e-mail: sh_bagirov@yahoo.com

Denote: $R^{+}=[0;+\infty)$, $B_R=\{x:|x|<R\}$, $B'_R=\{x:|x|>R\}$, $B_{R_1,R_2}=\{x:R_1<|x|<R_2\}$, $Q_T^{R_1,R_2}=B_{R_1,R_2}\times(0,T)$, $Q_T^{R,\infty}=B'_R\times(0,T)$, $Q_T=\Omega\times(0,T)$, $Q=\Omega\times(-\infty;+\infty)$, where Ω is the exterior of a compact set D in R_x^n containing the origin,.

Consider the equation

$$\frac{\partial u}{\partial t}=div(A(x,t)\nabla u)+h(x,t,u) \tag{1}$$

in the cylinder Q, where $A(x,t)=\left(a_{ij}(x,t)\right)_{ij=1}^{n}$, $h(x,t,u):\Omega\times(-\infty,+\infty)\times R^{+}\to R$, $n\geq 3$, $a_{ij}(x,t)$ are bounded, measurable, T- periodic in t functions, and there exist constants ν_1,ν_2 such that

$$\nu_1|\xi|^2\leq(A\xi,\xi)\leq\nu_2|\xi|^2 \tag{2}$$

for every $(x,t)\in Q,\xi=(\xi_1,...,\xi_n)\in R^n$. Here $\nabla u=\left(\frac{\partial u}{\partial x_1},...,\frac{\partial u}{\partial x_n}\right)$, $A\nabla u=\left(\sum_{j=1}^{n}a_{ij}\frac{\partial u}{\partial x_j}\right)_{i=1}^{n}$,

$(A\xi,\eta)=\sum_{i,j=1}^{n}a_{ij}\xi_i\eta_j$, $\xi=(\xi_1,...,\xi_n)$, $\eta=(\eta_1,...,\eta_n)$, $div(A\nabla u)=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}(a_{ij}(x,t)\frac{\partial u}{\partial x_j})$.

We will study the existence of a global positive solution of equation (1). The matters of existence and non-existence of global solutions for different classes of differential equations and inequalities play an important role both in theory and applications, that is why they have always been a cause for constant interest of mathematicians. Interest in such problems arose after paper [1]. After this work, many authors began to investigate the question of the existence of global solutions of various types of equations with non-linearity's of various types.

A solution of equation (1) is defined as a function $u(x,t)\in W_{2,loc}^{1,1/2}(Q_T)\cap L_{\infty,loc}(Q_T)$ satisfying the corresponding integral identity

$$2\pi\sum_{k=-\infty}^{k=+\infty}ik\int_{\Omega}u_k(x)\varphi_{-k}(x)dx+\int_{Q}(A(x,t)\nabla u,\nabla\varphi)dxdt=\int_{Q}h(x,t,u)dxdt$$

for each function $\varphi(x,t)\in \overset{0}{W}{}_2^{1,\frac{1}{2}}$.

We assume that $h(x,t,u) \geq \tilde{h}(x,u) \geq 0$ for all $(x,t) \in Q$ and $\tilde{h} : \Omega \times R^{+} \to R^{+}$ is a function satisfying the following.

(H):

a) for any $x \in B'_e$

$$\frac{\tilde{h}(x,s_1)}{s_1} \geq \frac{\tilde{h}(x,s_2)}{s_2} \quad \text{if } s_1 \geq s_2 > 0,$$

b) for any $\tau > 0$,

$$\lim_{|x| \to +\infty} \inf \tilde{h}(x, \tau |x^{2-n}|)|x|^n > C_0,$$

If b) fails, we assume that

b1) there exists $\sigma_1 \in (0,1)$ such that for any $\tau > 0$

$$\lim_{|x| \to +\infty} \inf \tilde{h}(x, \tau |x|^{2-n})|x|^n (\ln|x|)^{\sigma_1} > 0,$$

or

b2) there exists $\gamma > 1$ such that for any $\tau > 0, \alpha \geq 0$

$$\lim_{|x| \to +\infty} \inf \tilde{h}(x, \tau |x|^{2-n} (\ln|x|)^{\alpha})|x|^n (\ln|x|)^{-\alpha\gamma+1} > 0,$$

and

b3) there exists $\sigma_2 > 0$ such that for any $\tau > 0$,

$$\lim_{|x| \to +\infty} \inf \frac{\tilde{h}(x, \tau |x|^{2-n} (\ln|x|)^{\sigma_2})}{\tau |x|^{-n} (\ln|x|)^{\sigma_2}} > C_0.$$

The main result is the following theorem.

Theorem. *Let* $n \geq 3$, $A(x,t)$ *satisfies the condition (2). Then under the assumption (H) equation (1) has no positive solution in* Q.

References

[1] H.A.Levine, The role of critical exponents in blowup theorems. SIAM Review, 32 (1990), no. 2, 262-288.

Bağırov Şirmayıl Həsən oğlu, Bakı Dövlət Universiteti, Diferensial və inteqral tənliklər kafedrası, tel: +99450 --- -- --