

**MINISTRY OF SCIENCE AND EDUCATION  
REPUBLIC OF AZERBAIJAN  
BAKU STATE UNIVERSITY  
PROGRAM  
FOR MASTER'S DEGREE**

**SPECIALTY: 060509 COMPUTER SCIENCES  
SPECIALIZATION: MATHEMATICAL MODELING**

**SUBJECT: IF-M5328 MATHEMATICAL MODELING OF  
PETROLEUM MECHANICS PROBLEMS**

Approved as a subject program by the decision of the meeting  
№02 of the Scientific Council of the Faculty of Applied  
Mathematics and Cybernetics of Baku State University dated  
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**( III semester)**

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# **MATHEMATICAL MODELING OF PETROLEUM MECHANICS PROBLEMS**

## **Explanation sheet**

Petroleum and gas, which play a huge role in the country's economy, are the largest energy carriers among natural resources, so they play an efficient and leading role in energy. One of the areas where mathematical modeling techniques are widely used is the petroleum and gas extraction area. It is known that the production of petroleum and gas can begin with exploration work in the reservoir and drilling of exploration wells, their extraction, accumulation, transportation, storage through operational wells, etc. it involves complex technological processes. The development and operation of petroleum and gas wells, being a system, differs from other physical, technical systems in that it is extremely complex. The most important feature of these systems is that the processes taking place in them cannot be directly observed and repeated. Since it is possible to operate oil and gas layers only once, it is impossible to conduct numerous experiments on these systems and correct mistakes made in the process of their operation. Therefore, rational exploitation of oil and gas reservoirs leads to the need to use mathematical and computer modeling methods. Mathematical models of fluid flow flows occurring in the process of non-functioning of formation systems are described by complex special derivative differential equations. However, it is not possible

to establish the solution of these equations by analytical methods and to study fluid flows on their basis. Therefore, the technology of computer investigation of mathematical models of fluid flows is used. In order to learn these methods, it is considered necessary to teach Mathematical modeling of petroleum mechanics problems **IF-M5328** in accordance with state standards of Higher Education.

In the autumn semester of the II Year of study in the direction of Mathematical modeling of the specialty 060509- Computer science of the master's degree in this subject 45 hours (30 p. muh., 15 p. mesh.) is held in volume.

In the teaching of the subject

**purpose:**

- construction of mathematical models of physical processes occurring in petroleum reservoirs;
- study of methods of computer implementation of mathematical models;
- develop the ability to obtain information about the process according to the model;
- applying mathematical physics techniques to petroleum mechanics issues.

As a result of mastering the subject, undergraduates should

**know:**

- methods of constructing some mathematical models of petroleum mechanics;

- methods of mathematical physics applied to the problems formed when constructing models;
- numerical methods applied to the constructed discrete models;

**must be able to:**

- to build mathematical models of simple problems related to petroleum and gas dynamics
- to apply mathematical physics techniques when solving specific problems;
- to apply numerical methods of solution to discrete problems;

**must master:**

methods of constructing mathematical models of physical processes occurring in petroleum and gas reservoirs;

methods of computer implementation of mathematical models.

**Distribution of hours by topics**

	<b>The name of the subject</b>	<b>lect.</b>	<b>sem.</b>	<b>Sum.</b>
	<b>Topic № 1.</b> Stages of development of the petroleum and gas industry	2		2
	<b>Topic № 2.</b> Single-phase fluid flow	2		2
	<b>Topic № 3.</b> The role of petroleum and gas in the economy	2	2	4

	<b>Topic № 4.</b> Petroleum and gas extraction processes and mathematical modeling	2	2	4
	<b>Topic № 5.</b> Petroleum and gas reservoir development modes	2		2
	<b>Topic № 6.</b> Equations of motion of single-phase fluid flow	2	2	4
	<b>Topic № 7.</b> Equations of motion of single-phase fluid flow	2	2	4
	<b>Topic № 8.</b> Determinant equations of fluid and Layer	2		2
	<b>Topic № 9.</b> Mathematical models of single-phase fluid flow.	2	2	4
	<b>Topic № 10.</b> One-dimensional flow models	2		2
	<b>Topic № 11.</b> Settled Stream	2		2
	<b>Topic № 12.</b> Unsettled Stream	2	2	4
	<b>Topic № 13.</b> Numerical modeling methods of single-phase fluid flow in the Layer	2		2

	<b>Topic № 14.</b> Methods for discretizing a one-dimensional flow model of a single-phase fluid.	2	2	4
	<b>Topic 15.</b> Methods for numerical solution of the problem of one-dimensional flow of single-phase fluid.	2	1	3
		<b>30 h.</b>	<b>15 h</b>	<b>45 h.</b>

### **Subject content**

#### **1. Stages of development of the oil and gas industry**

History of oil discovery and chronology of its production in Azerbaijan. Famous oil scientists of Azerbaijan.

#### **2. Single-phase fluid flow**

Composition of oil. Physical properties of oil. Oil and gas formation.

#### **3. The role of petroleum and gas in the economy**

The largest oil and gas fields of Azerbaijan. "Contract of the century". Price of oil and gas as fuel and raw materials, areas of their application

#### **4. Petroleum and gas extraction processes and mathematical modeling**

Bringing the fluid flow formed in the process of developing layer systems to mathematical models - complex differential equations. Numerical model of fluid flow. Computer model of fluid flow.

### **5. Petroleum and gas reservoir development modes**

Water suppression mode. Gas suppression mode. Elastic mode. Dissolved gas mode. Gravitational mode.

### **6. Equation of discontinuity of single-phase fluid flow**

The laws of storage of mass, momentum and energy. Law of energy storage in fluid dynamics – energy equation. Isothermal flow. Derivation of the discontinuity equation for isothermal flow of a single-phase fluid based on the law of mass retention

### **7. Equations of motion of single-phase fluid flow**

Layer pressure and temperature. The formula for the relationship between the rate of filtration of single-phase fluid in the layer and the pressure gradient. Darcy's law for Newtonian-type fluids. Spelling of the equation of motion in the form of projections in the direction of the coordinate axes.

### **8. Determinant equations of fluid and layer**

The determinant equation of the formation fluid in the general case. The determining equation for the isothermal process. Determinant equation for incompressible and weakly compressible fluids. Determinant equations of sharply compressed fluids, ideal gas in isothermal process.

### **9. Mathematical models of single-phase fluid flow**

Mathematical model of isothermal flow obeying the Darcy law of a single-phase fluid in an isotropic layer. System of differential equations with special derivatives consisting of determinant equations of discontinuity, motion, fluid and layer

### **10. One-dimensional flow models**

Rectilinear parallel flow. Model of rectilinear parallel flow of weakly compressible fluid in deformable formation systems consisting of a one-dimensional parabolic equation. Initial and boundary conditions that can be set for this equation. Mathematical models of plane radial flow and radial spherical flow

### **11. Settled stream**

Settled rectilinear parallel flow. Mathematical model of rectilinear parallel flow of incompressible Newton-type fluid in a homogeneous layer in the form of a non-deformable rectangular parallelepiped. The relationship between pressure distribution and extraction. Settled plane radial flow. The dupe formula. Settled radial spherical flow. Determination of fluid filtration rate and well production according to reservoir pressure.

### **12. Unsteady flow**

Unsteady rectilinear parallel flow and the law of pressure distribution for it. Unsteady plane radial flow. Fluid filtration rate and pressure in the reservoir.

### **13. Numerical modeling methods of single-phase fluid flow in a layer**

Numerical model. Computer model. The main elements of the method of finite differences. Methods for discretization of differential issues. Construction of a discrete analogue of a specific problem. Approximation, resilience and accumulation.

**14. Methods for discretizing a one-dimensional flow model of a single-phase fluid.**

Construction of a discrete analogue of the equation describing the rectilinear parallel flow of a weakly compressed fluid in a deformable layer. Detection method. Non-obvious method. Crank-Nicholson method.

**15. Methods for numerical solution of the problem of one-dimensional flow of single-phase fluid..**

Discrete analogues of the problem of undetected rectilinear parallel flow of a weakly compressed fluid in a deformable layer in the form of a rectangular parallelepiped, constructed by obvious and non - obvious methods. Application of numerical methods to the solution of the obtained system of linear algebraic equations. Obtaining the stability conditions of the solution.

**List of literature**

**Basic**

Mirzəcanzadə A.X., Kərimov K.K. Neft və qaz yataqlarının işlənməsinin nəzəri əsasları. Bakı: Elm, 1972.

Мирзаджанзаде А.З., Халиулин В.Н. Основы теории разработки нефтяных месторождений. Москва: Недра, 1965.

Chen, Z., Huan, G. and Ma, Y. (2006) Computational Methods for Multiphase Flows in Porous Media. Society for Industrial and Applied Mathematics, 531 p.

Həmzəyev X.M. Lay sistemlərinin riyazi modelləşdirilməsi: Dərs vəsaiti. Bakı: ADNSU Nəşriyyatı, 2016.

Мирзаджанзаде А.З., Керимов З.Г., Копейкис М.Г. Теория колебаний в нефтепромысловом деле. Москва: Недра, 1977.

### **Additional literature**

Mirzəcanzadə A.X. Qazma quyularının nəzəriyyəsi. Bakı: ADNSU Nəşriyyatı, 1968.

Əliyev R.Ə., Əliyev M.Ə. Neft və qaz mühəndisliyinin əsasları. Bakı: Azərbaycan Dövlət Neft və Sənaye Universiteti Nəşriyyatı, 2004.

Marc D. Zoback Reservoir geomechanics Cambridge University Press, 2007), 505 p.

Экономидес М.Дж., Нолте К.Г. Стимуляция пласта. Москва: Недра, 2004 (оригинал: Wiley, 2000).

5. Articles and materials: Anas Earth Sciences series, Journal of oil and gas engineering. Baku: Anas, different years.